Decision Mathematics 2 Mark Scheme

Question	Scheme	Marks	AOs
1	$\begin{pmatrix} & P & Q & R & S & T & X \\ A & 32 & 32 & 35 & 34 & 33 & 40 \\ B & 28 & 35 & 31 & 37 & 40 & 40 \\ C & 35 & 29 & 33 & 36 & 35 & 40 \\ D & 36 & 30 & 34 & 33 & 35 & 40 \\ E & 30 & 31 & 29 & 37 & 36 & 40 \\ F & 29 & 28 & 32 & 31 & 34 & 40 \end{pmatrix}$	B1	1.1b
	Reducing rows and then columns $\begin{pmatrix} & P & Q & R & S & T & X \\ A & 0 & 0 & 3 & 2 & 1 & 8 \\ B & 0 & 7 & 3 & 9 & 12 & 12 \\ C & 6 & 0 & 4 & 7 & 6 & 11 \\ D & 6 & 0 & 4 & 3 & 5 & 10 \\ E & 1 & 2 & 0 & 8 & 7 & 11 \\ F & 1 & 0 & 4 & 3 & 6 & 12 \end{pmatrix}$ then $\begin{pmatrix} & P & Q & R & S & T & X \\ A & 0 & 0 & 3 & 0 & 0 & 0 \\ B & 0 & 7 & 3 & 7 & 11 & 4 \\ C & 6 & 0 & 4 & 5 & 5 & 3 \\ D & 6 & 0 & 4 & 1 & 4 & 2 \\ E & 1 & 2 & 0 & 6 & 6 & 3 \\ F & 1 & 0 & 4 & 1 & 5 & 4 \end{pmatrix}$	M1 A1	1.1b 1.1b
	e.g. augment by 1 then augment by 1 $\begin{pmatrix} & P & Q & R & S & T & X \\ A & 1 & 1 & 3 & 0 & 0 & 0 \\ B & 0 & 7 & 2 & 6 & 10 & 3 \\ C & 6 & 0 & 3 & 4 & 4 & 2 \\ D & 6 & 0 & 3 & 0 & 4 & 1 \\ E & 2 & 3 & 0 & 6 & 6 & 3 \\ F & 1 & 0 & 3 & 0 & 4 & 3 \end{pmatrix}$ followed by $\begin{pmatrix} & P & Q & R & S & T & X \\ A & 2 & 2 & 3 & 1 & 0 & 0 \\ B & 0 & 7 & 1 & 6 & 9 & 2 \\ C & 6 & 0 & 2 & 4 & 3 & 1 \\ D & 6 & 0 & 2 & 0 & 3 & 0 \\ E & 3 & 4 & 0 & 7 & 6 & 3 \\ F & 1 & 0 & 2 & 0 & 3 & 2 \end{pmatrix}$	M1 A1ft M1 A1ft A1	1.1b 1.1b 1.1b 1.1b 1.1b
	A – T, B – P, C – Q, (D –), E – R, F – S	A1	2.2a
			(9 marks)

Notes:

B1: cao – introducing a dummy task and appropriate value
M1: Simplifying the initial matrix by reducing rows and then columns
A1: cao
M1: Develop an improved solution – need to see Double covered +e; one uncovered –e ; and one single covered unchanged. 4 lines to 5 lines needed
A1ft: ft on their previous table – no errors
M1: Finding the optimal solution – need to see one double covered +e; one uncovered –e; and one single covered unchanged. 5 lines needed to 6 lines needed (so getting to the optimal table)
A1ft: ft on their previous table – no errors
A1: cso on final table (so must have scored all previous marks)
A1: cso – this mark is dependent on all M marks being awarded – to deduce the optimal allocation from the location of zeros in the table

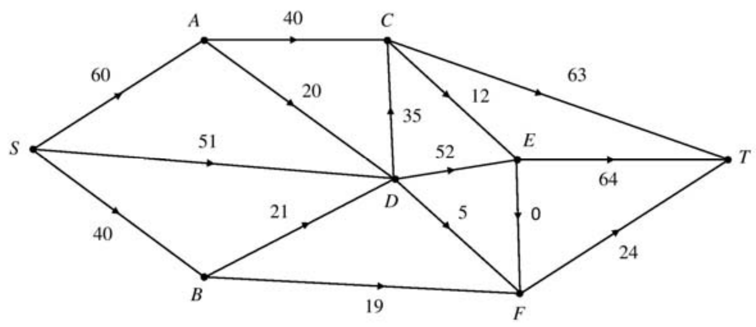
Question	Scheme	Marks	AOs
2(a)	16, 22, 29	B1	1.1b
		(1)	
(b)	$u_{n+1} = u_n + n + 1$	B1	3.3
		(1)	
(c)	As $u_{n+1} = u_n + p(n) \Rightarrow u_n = \lambda n^2 + \mu n + \phi$ and attempt to solve with $n = 1, 2, 3$	M1	1.1b
	$u_n = \frac{1}{2}n(n+1)+1$	A1	1.1b
	20 101 (regions)	A1ft	1.1b
		(3)	
			(5 marks)

Notes:

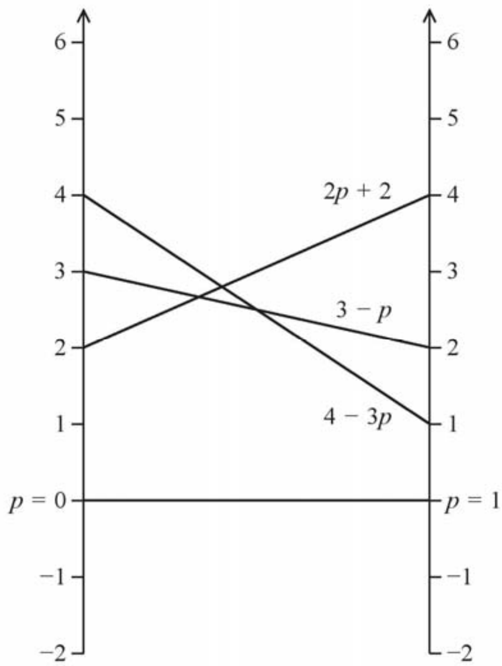
(a)
B1: cao

(b)
B1: Translating problem to mathematical model - correct recurrence relation needed

(c)
M1: An attempt to solve the recurrence relation to determine maximum number of regions
A1: cao
A1ft: Substitution of n = 200 into their quadratic u_n expression

Question	Scheme	Marks	AOs
3(a)	Corridors must be one-way	B1	3.4
		(1)	
(b)	e.g. $55 + x + 40 = 63 + 54 + 24$ **or** $7 + y = 54 + 5$	M1	2.4
	$x = 46$ $y = 52$	A1 A1	1.1b 1.1b
		(3)	
(c)	**(i)** SACET (= 5) SDFET (= 5)	M1 A1	1.1b 1.1b
	(ii) Students must choose SACET, as they cannot travel from F to E	A1	2.2a
		(3)	
(d)		B1	1.1b
		(1)	
(e)	Use of max-flow min-cut theorem	M1	2.1
	Identification of cut through AC, DC, DE, (EF), FT = 151 value of flow = 151	A1	3.1a
	Therefore it follows that flow is optimal	A1	2.2a
		(3)	
(f)	Consider increasing capacity of arcs in minimum cut	B1	2.1
	Explanation based on a valid argument, such as: • increasing the capacity of any arc other than FT would not increase the flow by more than 1, as total capacity directly in to T is only 152 • increasing the capacity on FT could increase the total flow by 16 (increased flow along SAD, SD and SBD could all be directed through DF to F)	B1	2.4
	Therefore school should choose to widen FT, which could increase the flow through the network by 16	B1	2.2a
		(3)	
			(14 marks)

Question **3** notes:	
(a)	
B1:	Explanation of assumption to use this model
(b)	
M1:	Either a correct equation, or explanation that flow in = flow out
A1:	cao
A1:	cao
(c)	
M1:	One flow augmenting route found from S to T
A1:	Two correct flow augmenting routes 5+
A1:	Deduce that SACET must be used as students cannot travel from F to E as route is one-way
(d)	
B1:	A consistent flow pattern = 151
(e)	
M1:	Constructing argument based on max-flow min-cut theorem
A1:	Use appropriate process of finding a minimum cut – cut + value correct
A1:	Correct deduction that the flow is maximal
(f)	
B1	Constructing an argument based on arcs in the minimum cut
B1	Detailed explanation as to why choosing anything other than FT does not help
B1	Correct deduction and correct increase in flow of 16

Question	Scheme	Marks	AOs
4(a)	Row minima: 1, 2 max is 2 Column maxima: 4, 4, 3 min is 3	M1 A1	1.1b 1.1b
	Row maximin (2) $\neq$ Column minimax (3) so not stable	A1	2.4
		(3)	
(b)	Let A play strategy 1 with probability p and strategy 2 with probability 1- p, and using this to get at least one equation in p	M1	3.3
	Then if B plays strategy 1, A's gains are $4p + 2(1-p) = 2p + 2$ If B plays strategy 2, A's gains are $p + 4(1-p) = 4 - 3p$ If B plays strategy 3, A's gains are $2p + 3(1-p) = 3 - p$ 	A1 A1	1.1b 1.1b
	Intersection of $2p + 2$ and $3 - p$ occurs where $p = \frac{1}{3}$	dM1 A1ft	1.1b 1.1b
	Therefore player A should play strategy 1 $\frac{1}{3}$ of the time and play strategy 2 $\frac{2}{3}$ of the time	A1ft	3.2a
	The value of the game to player A is $2\frac{2}{3}$	A1	1.1b
		(9)	
		(12 marks)	

Question **4** notes:	
(a)	
M1:	Finding row minimums and column maximums – condone one error
A1:	Row minima and column maxima correct
A1:	Explanation involving $2 \neq 3$ and a conclusion
(b)	
M1:	Translating situation into model by defining variables and constructing at least one equation
A1:	One row correct
A1:	All three rows correct
M1:	Axes correct, at least one line correctly drawn for their expression
A1:	Correct graph
M1:	Using their probability expectation graph to find the probability by equating their two correct expressions and attempting to solve as far as $p =$
A1ft:	ft on their optimal intersection
A1ft:	Interpret their value of p in the context of the question – must refer to play, player A
A1:	cao

